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Seminar - Special Aspects of Insurance Mathematics

Negative Correlation between Gompertz Parameters

Ivy Woo | February 9, 2020 | Institute of Insurance Science

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Mortality Models

- Gompertz Model (GM):

$$\mu(t) = \mu_0 e^{\gamma t}$$

- Gomertz-Makeham Model (GMM):

$$\mu(t) = C + \mu_0 e^{\gamma t}$$

- Parameters' meaning:
  - μ_0 : initial resistance to causes of deaths / initial vitality
  - γ : rate of age-dependent decline in vitality
  - C : age-independent component of total mortality (Makeham component)

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Paper

- Strehler, B. L., & Mildvan, A. S. (1960). General theory of mortality and aging. Science, 132(3418), 14-21.
- Proposes a negative linear relationship between $\log \mu_0$ and γ from the Gompertz model
- Since then such negative correlation is termed as the Strehler-Mildvan Correlation (SMC)

Terminologies

- Vitality $V(t)$:
  Capacity of an organism to stay alive at certain time t.
- Attrition coefficient $B(t)$:
  The fractional loss each year of original vitatlity V_0, which is the sum of attritions due to normal aging b and environmental factors $f(D(t))$, i.e.

$$V(t) = V_0\left[1 - B(t) \cdot t\right] \text{ and } B(t) = b + f(D(t))$$

  $D(t)$: some measure of the relative deleteriousness of an environment.

Terminologies

- Maxwell-Boltzmann distribution:
  A distribution used in Physics for describing particle speeds in idealized gases

$$\frac{n}{n_t} = k\left(\frac{E}{RT}\right)^{\frac{1}{2}} e^{-\frac{E}{RT}} =: Ke^{-\frac{E}{RT}}$$

k : Boltzmann constant
T : absolute temperature, R : gas constant
E : energy of a state of molecular system
n : number of molecules with energy $\geq E$
n_t : total number of molecules

Postulates

1. An organism has a certain maximum capacity to restore initial conditions after a “challenge”.
   Death occurs when rate to restore original state is less than that demanded to overcome the challenge.

$$\mu(t) = C \cdot X(t)$$

C : total number of challenges per unit time

$X(t)$: fraction of challenges greater than or equal to $V(t)$

2. Magnitude of challenges follows the Maxwell-Boltzmann distribution.

$$X(t) = K' e^{\frac{-V(t)}{\epsilon D(t)}}$$

where $K' = \frac{K}{C}$, ϵ is some constant such that $\frac{-V(t)}{\epsilon D(t)}$ is unitless

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Strehler-Mildvan Correlation

- Suppose mortality follows the Gompertz law $\mu(t) = \mu_0 e^{\gamma t}$. After manipulating terms, one finds $B(t) = B$ is constant and gets

$$\log \mu_0 - \log K = -\gamma / B$$

  $\Rightarrow$Implies linear negative correlation between $\log \mu_0$ and γ

- A linear negative correlation is observed from analysis on human data, which supports the proposed theory

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Paper

- Golubev, A. (2019). A 2D analysis of correlations between the parameters of the Gompertz–Makeham model (or law?) of relationships between aging, mortality, and longevity. Biogerontology, 20(6), 799-821.
- By visualizing the Gompertz parameters in a semi-log graph, termed as the “Gompetz Model 2D (GM2D) Plot”, the author argues what the reasons for the negative correlation between parameters are.

Recent Propositions

- Possible reasons for the SMC:
  - Due to noisiness of data / sampling error, "degenerate manifold of Gompertz fit" [1]
  - Changes in the age-independent component C [2,3]

  } Artifacts

  - Compensation Effect of Mortality (CEM):
    Real trade-off between initial vitality and rate of decline in vitality

1 Tarkhov, A. E., Menshikov, L. I., & Fedichev, P. O. (2017). Strehler-Mildvan correlation is a degenerate manifold of Gompertz fit. Journal of theoretical biology, 416, 180-189.

2 Gavrilov, L. A., & Gavrilova, N. S. (1991). The biology of life span: a quantitative approach. New York/ Chur Switzerland Harwood Academic, New York.

3 Gavrilov, L. A., & Gavrilova, N. S. (2001). The reliability theory of aging and longevity. Journal of theoretical Biology, 213(4), 527-545.

Reference lines

- Isoage line:
  all pairs of μ_0 and γ on the same isoage line imply the same life expectancy under GM [1]

- SMC line:
  regression line from applying GM to data generated by GMM upon varying C
  $\Rightarrow$the negative correlation that would be observed when there is no change in μ_0 and γ but a change in C

1 Life expectancy from Gompertz model (Tarkhov et al. (2017)):
Mean age $= \frac{\exp(e^{\Lambda})}{\gamma}\left(-0.577 + \Lambda - \sum_{k=1}^{\infty} \frac{(-e^{-\Lambda})^k}{k(k!)}\right)$ where $\Lambda = \log \frac{\gamma}{\mu_0}$

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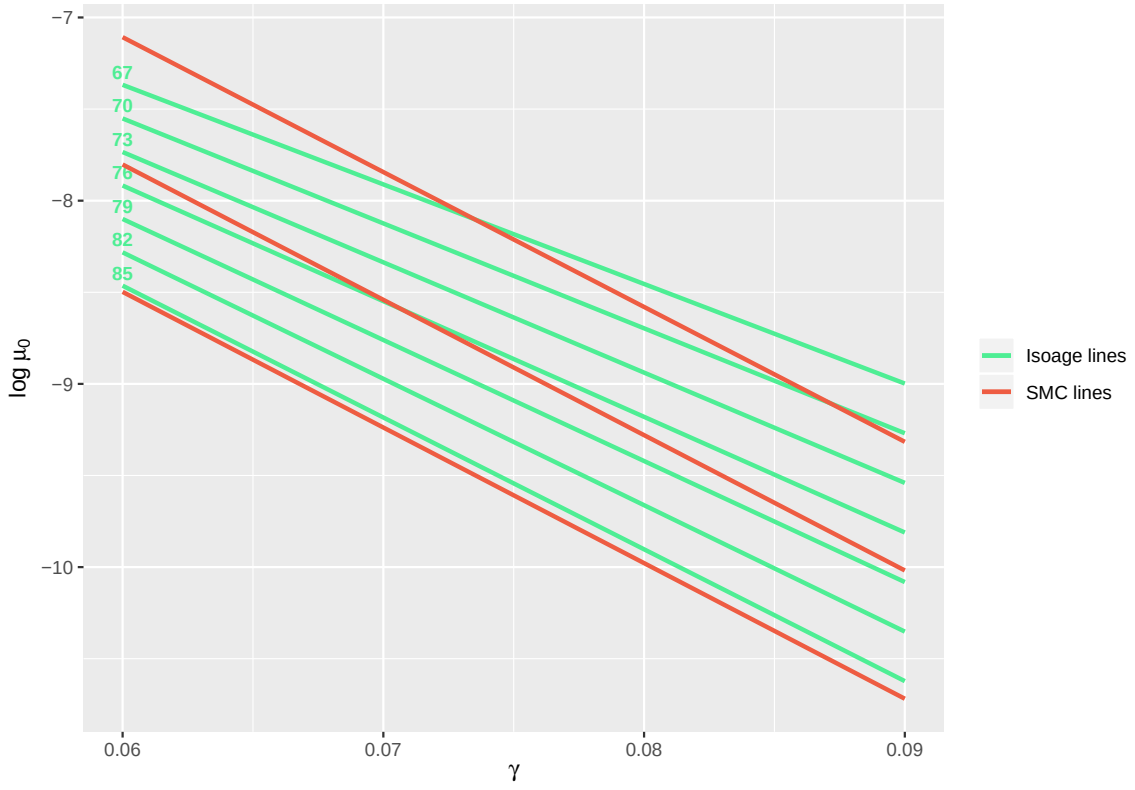
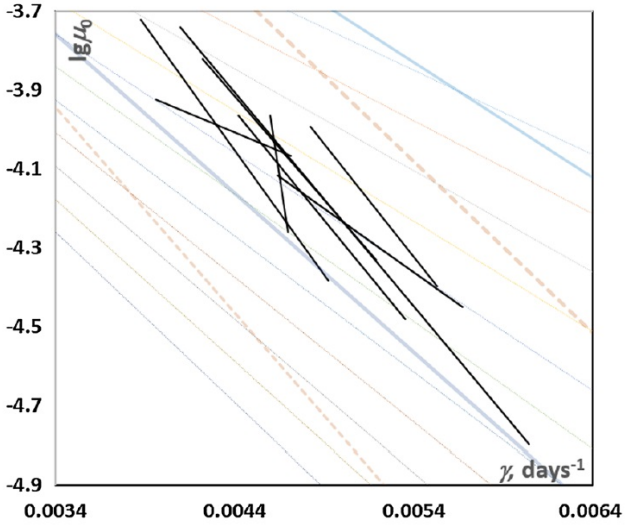
Illustration of reference lines



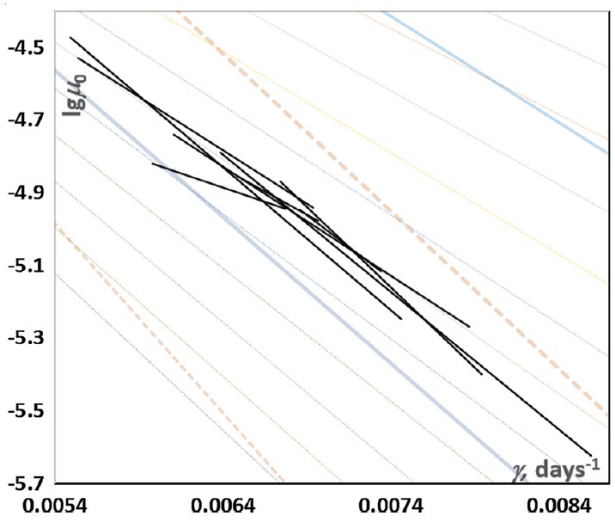
- SMC line slope $>$ Isoage line slope (in absolute term)

Analysis - Examples

Golubev (2019) figures 7A and 7B :



(a) Regression on male samples



(b) Regression on female samples

- Male: slope of regression $>$ SMC line slope in most cases
  $\Rightarrow$CEM is implied, negative correlation is not artifact
- Female: slope of regression $\approx$ isoage line slope
  $\Rightarrow$Nothing more than artifact is seen

Analysis - Examples

Golubev (2019) figure 8(JL) :

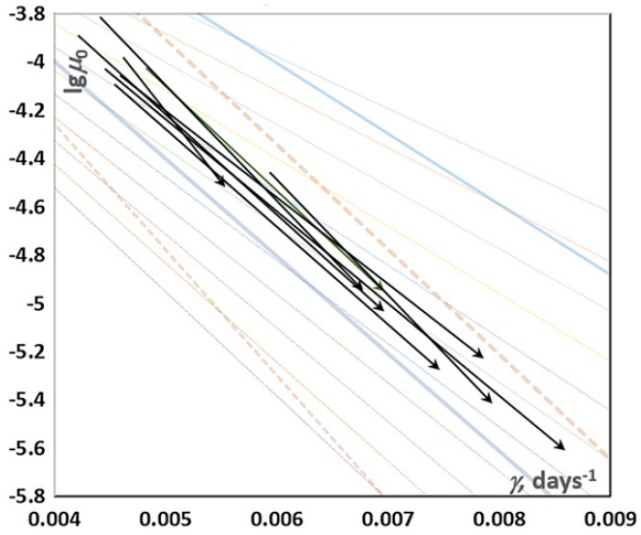


Figure: Vectors directing from males to females

- All vectors are directing from top left to bottom right (which is not implied by the artifacts of Gompertz fit)

$\Rightarrow$ Implies CEM is in effect: females feature higher initial vitality which is associated with its faster rate of decline

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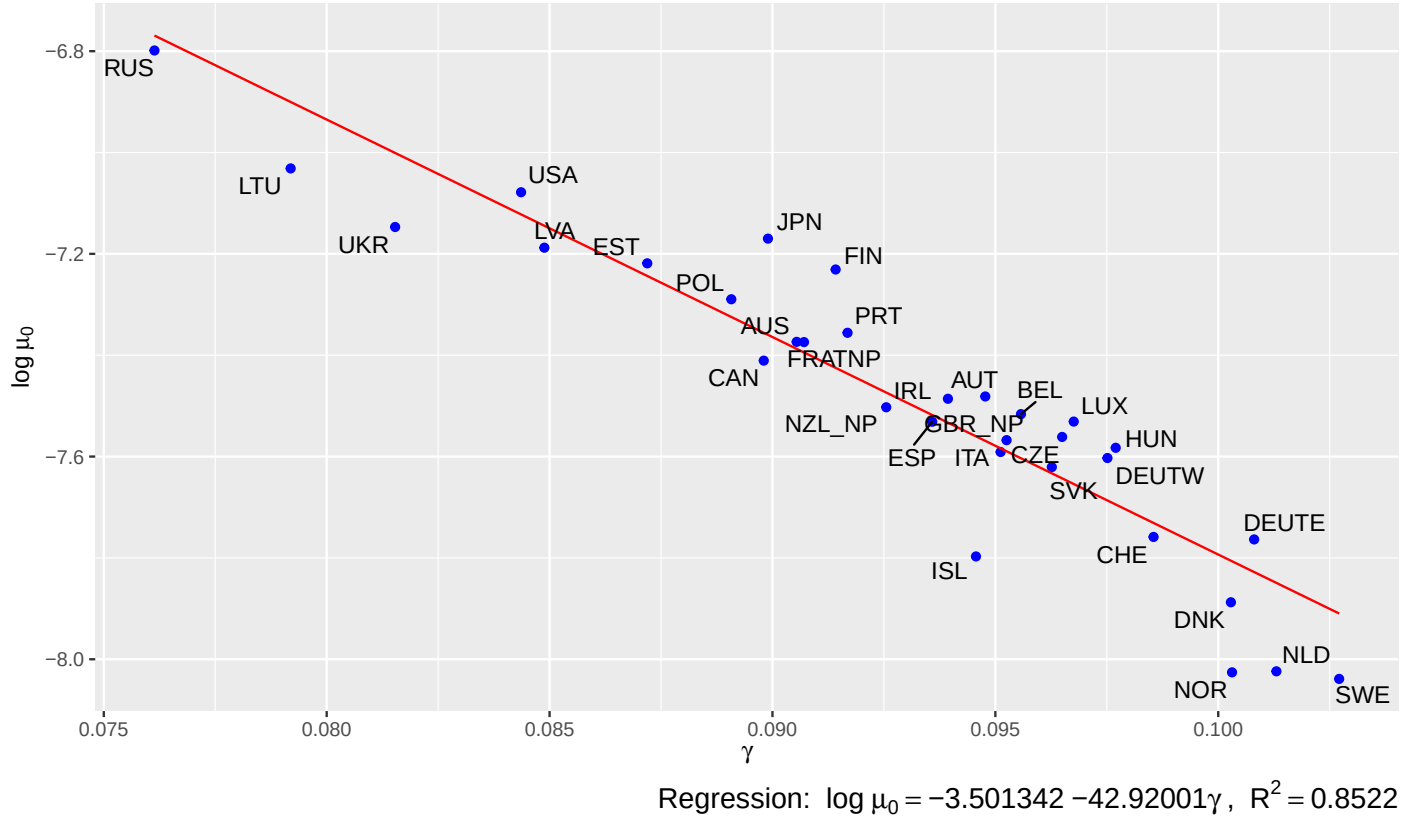
Human Mortality

- In Golubev's paper:
  - "Rectangularization" is observed in human mortality experience according to other literature
  - CEM is proposed to be a possible reason for this
- We do the same kind of analysis on human mortality data from the HMD website

Data

- R package “MortalityLaws”
  - downloads data from HMD website
  - estimates μ_0 and γ by minimizing certain loss function, here chosen to be
    $-[Dx \cdot \log(\text{estimated value}) - Ex \cdot \text{estimated value}]$
    where $Dx =$ Death counts, $Ex =$ Population exposed to risk
- Age range 25-90
- 32 countries from Europe, North America, Asia and Australia

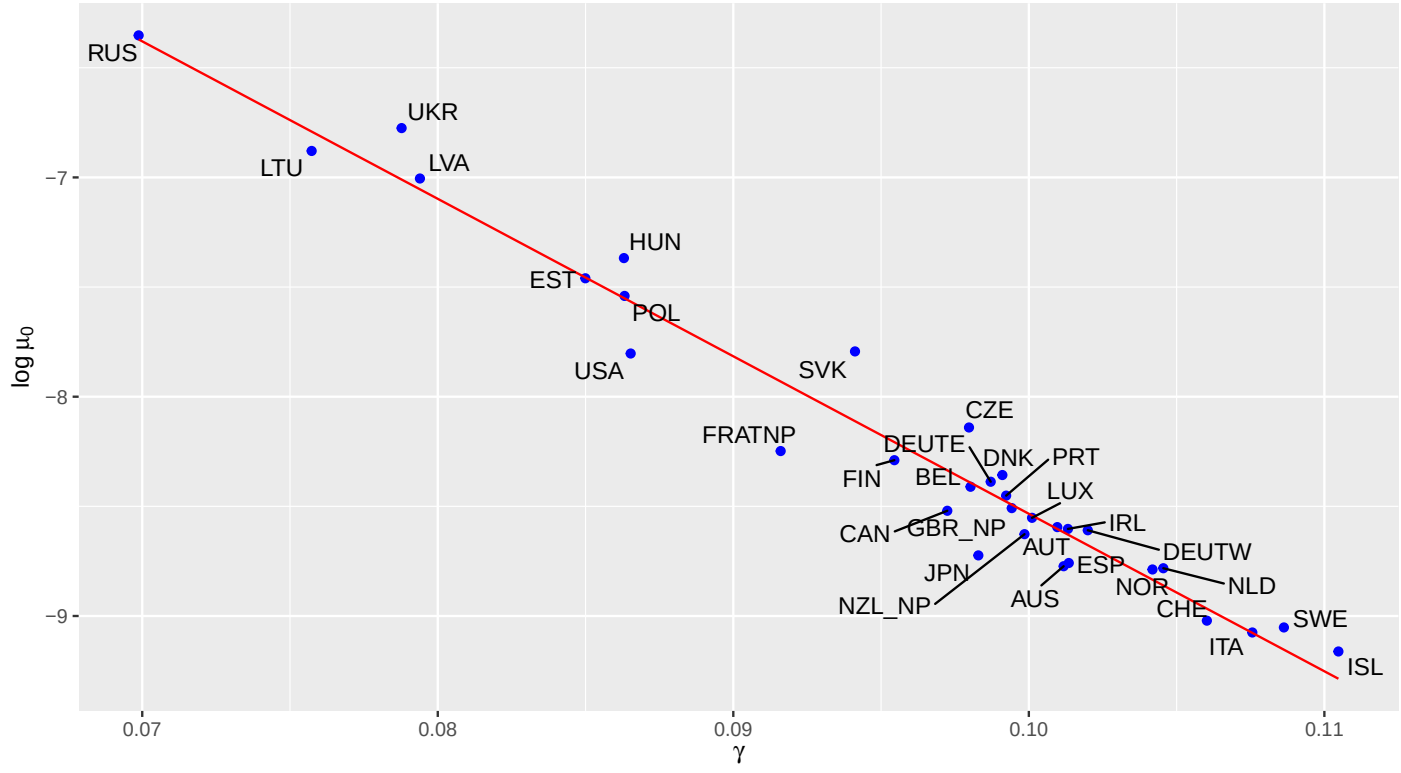
GM2D plot for 32 countries in 1960



Regression: $\log \mu_0 = -3.501342 - 42.92001\gamma$, $R^2 = 0.8522$

- Negative correlation observed

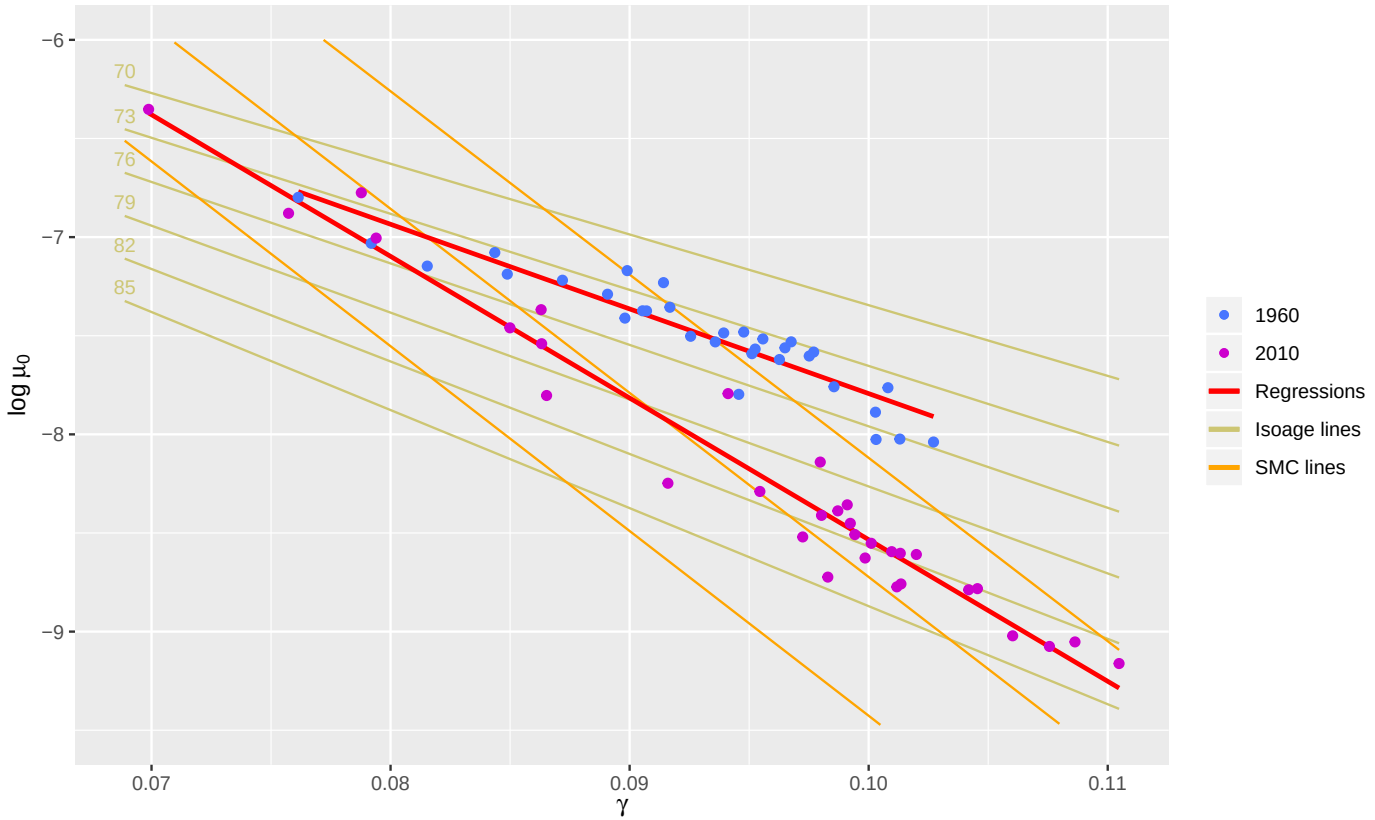
GM2D plot for 32 countries in 2010



Regression: $\log \mu_0 = -1.35245 - 71.8148\gamma$, $R^2 = 0.9577$

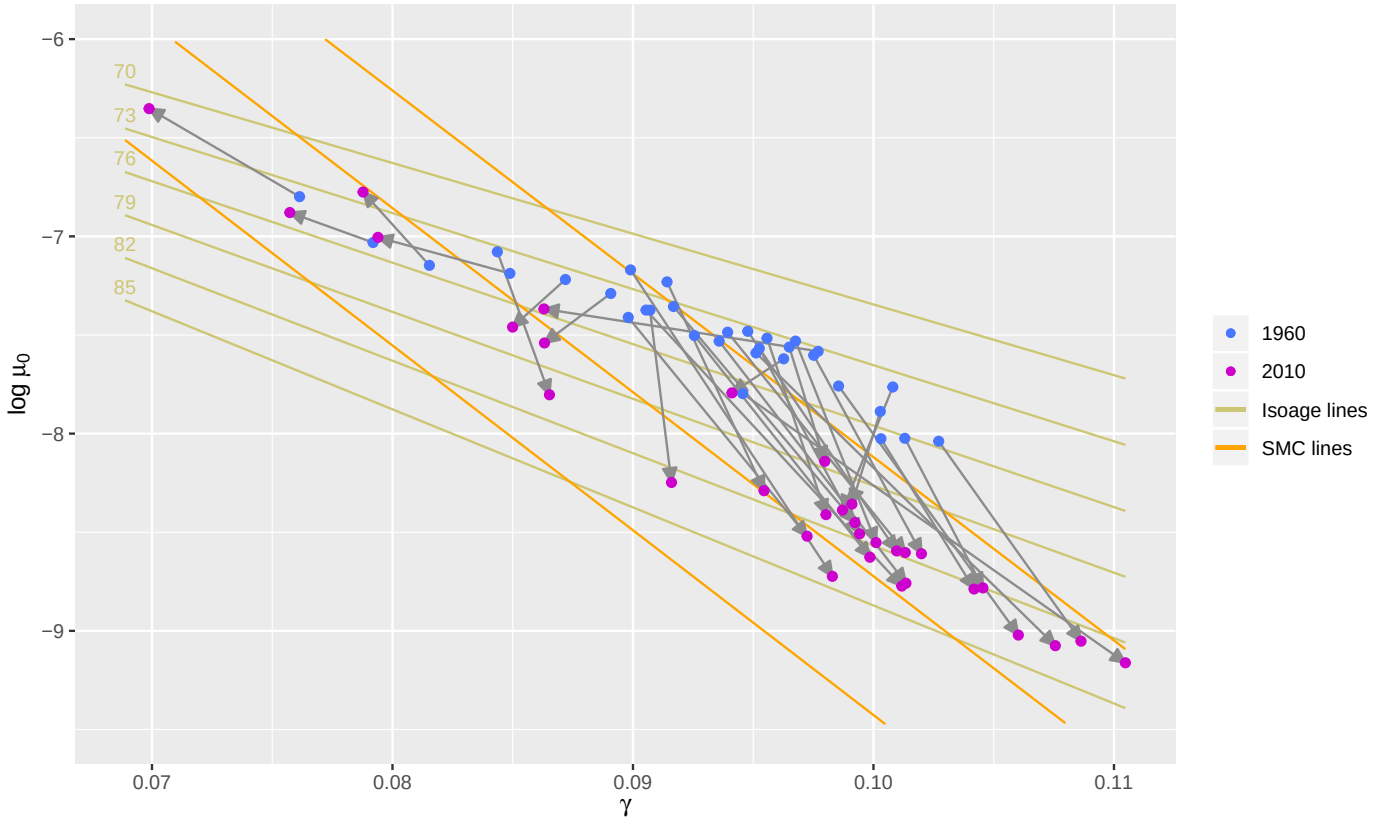
- Negative correlation observed
- Slope becomes steeper

Comparison from 1960 to 2010



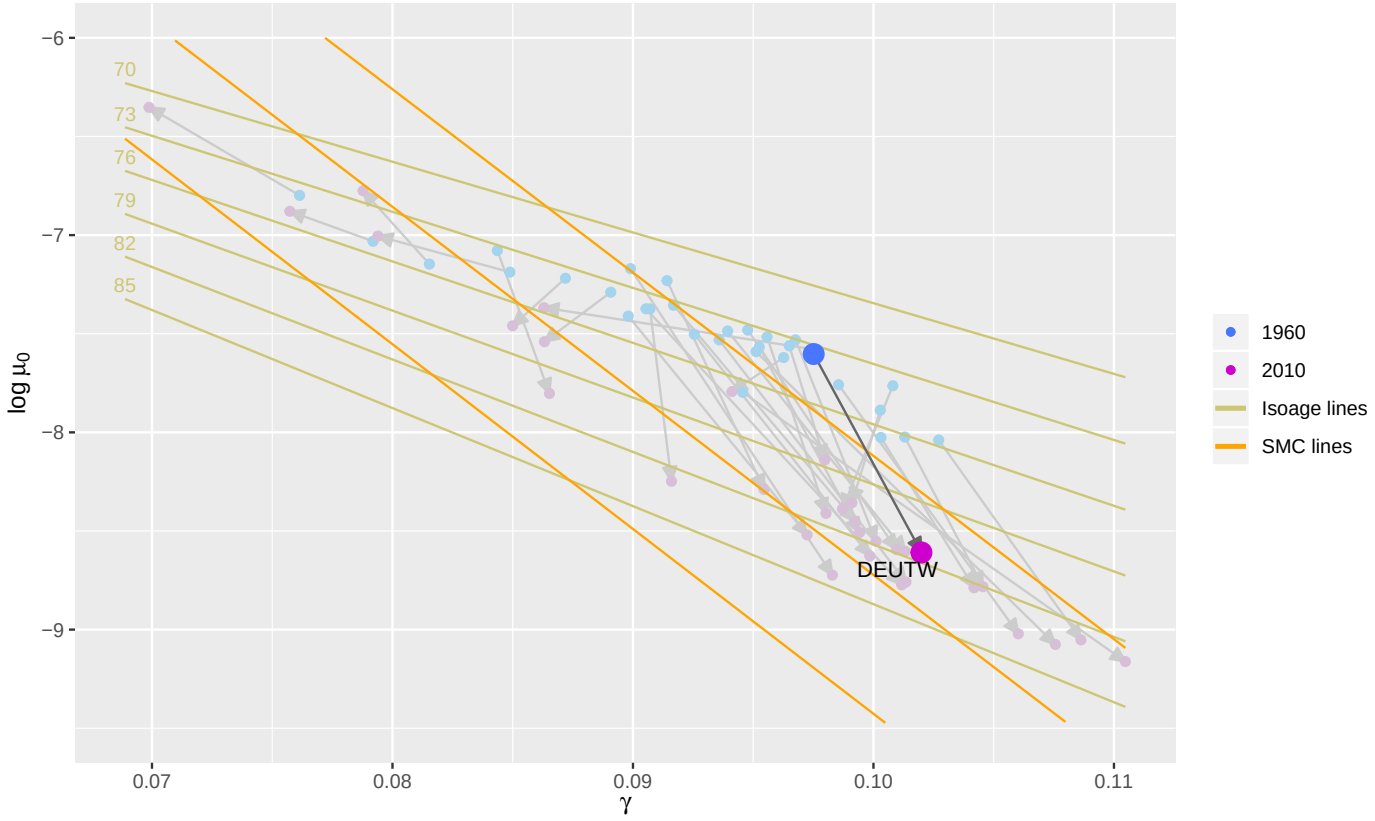
- Significant improvement in life expectancy
- 1960 : regression slope $\approx$ isoage slope
  2010 : isoage slope $<$ regression slope $<$ SMC slope

Comparison from 1960 to 2010



- Change is not unidirectional

West Germany from 1960 to 2010



- Life expectancy increases from 73 to 82
- Rightward-downward trend with slope $>$ SMC line slope
  $\Rightarrow$CEM is implied for West Germany

Thank you!